First derivative Test: Suppose that f is continuous on an interval that contains a critical point c, and assume that f is differentiable on an interval containing c, except perhaps at c itself.

- If f' changes sign from positive to negative as x increases through c, then f has a local maximum at c.

  f' $-\infty \xrightarrow{\quad c \quad} +\infty$ f has a local max

- If f' changes sign from negative to positive as x increases through c, then f has a local minimum at c.

  f' $-\infty \xrightarrow{\quad c \quad} \infty$ f has a local min

- If f' does not change sign at c, then f has no local extreme value at c.

Eg. $f(x) = x^2+3$ in $[-3, 2]$

$f'(x) = 2x$, critical point $x=0$

$f'(-1) = -2$; $f'(1) = 2$

Hence f has a local min at $x=0$, minimum value $f(0) = 3$ //

$\frac{\sqrt{x}}{x} = \frac{\sqrt{x}}{\sqrt{x}\sqrt{x}}$

Eg $f(x) = \sqrt{x}\ \ln x$, $(0, \infty)$

$f'(x) = \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} = \frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}} = \frac{2+\ln x}{2\sqrt{x}}$; defined everywhere on $(0,\infty)$

$f'(x) = 0 \Rightarrow 2+\ln x = 0 \Rightarrow x = e^{-2} = \frac{1}{e^2}$ critical point.

$f'(1) > 0$; $f' < 0$ in $(0, \frac{1}{e^2})$, $f' > 0$ in $(\frac{1}{e^2}, \infty)$

$f(x)$ has only one critical point at $x = \frac{1}{e^2}$, this is a local minimum, value $f(\frac{1}{e^2})$. It has no absolute maximum since $f \to \infty$ as $x \to \infty$